Summary of Convergence Tests

Name	Statement	Comments
***N* th Term Test for Divergence**	If $\lim_{n\to\infty} u_n \neq 0$, then $\sum u_n$ diverges.	If $\lim_{n\to\infty} u_n = 0$, then $\sum u_n$ may or may not converge. The test fails.
Ratio Test	Let $\sum u_n$ be a series with positive terms and suppose that $\lim_{n\to\infty} \frac{u_n + 1}{u_n} = L$ a) The series converges if $L < 1$. b) Series diverges if $L > 1$ or $L = \infty$ c) The test is <u>inconclusive</u> if $L = 1$	Try this test when u_k involves factorials or *k*th powers.
Ratio Test for Absolute Convergence	Let $\sum u_n$ be a series with nonzero terms such that $\lim_{n\to\infty} \frac{\lvert u_n + 1\rvert}{\lvert u_n\rvert} = L$ a) The series converges absolutely if $L < 1$. b) The series diverges if $L > 1$ or $L = \infty$ c) The test is <u>inconclusive</u> for $L = 1$.	The series need not have positive terms and need not be alternating to use this test.
Direct Comparison Test (DCT)	Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be series with nonnegative terms such that $a_1 \leq b_1$, $a_2 \leq b_2$, ... , $a_k \leq b_k$, ... If $\sum b_n$ converges, then $\sum a_n$ converges, and if $\sum a_n$ diverges, then $\sum b_n$ diverges.	This test only applies to series with nonnegative terms. Try this test as a last resort; other tests are often easier to apply.
Limit Comparison Test (LCT)	Let $\sum a_n$ and $\sum b_n$ be series with positive terms such that $\lim_{n\to\infty} \frac{a_n}{b_n} = L$. If $0 < L < \infty$, then both series converge or both diverge.	This is easier to apply than the comparison test, but still requires some skill in choosing the series $\sum b_n$ for comparison.
Integral Test	Let $\sum u_n$ be a series with positive terms, and let $f(x)$ be the function that results when k is replaced by x in the general term of the series. If f is decreasing and continuous for $x \geq a$, then $\sum_{k=1}^{\infty} u_k$ and $\int_a^{\infty} f(x)dx$ both converge or both diverge.	This test only applies to series that have positive terms. Try this test when $f(x)$ is easy to integrate.
***p* - Series Test**	$\sum \frac{1}{n^p}$ The series converges if $p > 1$ and diverges if $p \leq 1$.	Often used in conjunction with a compatison test (DCT or LCT).
Alternating Series Test	Given $\sum_{n=1}^{\infty} (-1)^{n+1} u_n$, the series converges if the following 3 conditions hold true: a) $u_n > 0$ b) $u_1 \geq u_2 \geq u_3 \geq \ \ldots$ c) $\lim_{n\to\infty} u_n = 0$	This test applies only to alternating series. This test is inconclusive if any of these conditions does not hold true.
Root Test	Let $\sum u_n$ be a series with positive terms such that $\lim_{n\to\infty} \sqrt[n]{u_n} = L$ a) The series converges if $L < 1$. b) The series diverges if $L > 1$ or $L = \infty$ c) The series is inconclusive for $L = 1$	Try this test when u_n involves *n*th powers.